APPLICABILITY OF APPROXIMATE MODELS IN CALCULATING THE BOUNDARY REGION IN DUST-LADEN FLOWS

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The limits of application of "single- particle" and "quasihomogeneous liquid" models are established on the basis of analysis of Couette flow for a dust-laden gas.

The rate of transfer processes in flows of gas suspensions is determined largely by hydromechanical and thermal conditions in the boundary region of the flow. Radial transport of particles due to pulsative motion of the carrying medium, particle collisions, and various forces acting on the particles [1] in this region may have significant effects in terms of particle deposition on the walls, convective heat transfer, etc. The complexity of analytically describing the boundary region of gas suspension flows often makes it necessary either to assume that the particles have no effect in the characteristics of the continuous medium [2, 3] or to represent the entire flow in the form of a certain quasi-liquid with effective parameters. Obvoiusly, such approaches can be used only within a limited range of values of the determining parameters. For example, studies conducted earlier made it possible to determine the limits of application of a "quasihomogeneous liquid" approximation for convective heat exchange with gas suspension flows [4], heating of a gas suspension by powerful radiant flows [5], and complex heat exchange [6].

In the present work, features of friction and heat transfer are examined with reference to the boundary zone of dust-laden flows for different variants of transverse motion of the components (transverse motion of the gas with particles, transverse motion of the particles only, from and to the wall).

The two-phase Couette flow depicted in Fig. 1 will be used for the model of the boundary region of the gas suspension flow. In using this model, longitudinal flow is usually assumed to be nongradient. It may also be assumed that momentum and heat transfer due to particle collisions are negligibly small, and that the physical properties of the components are independent of temperature. The initial system of equations of motion and energy of the gas suspension flow under these conditions have the following form for viscous flow:

$$v\frac{du}{dy} + \mu v_t \frac{du_t}{dy} = \nu \frac{d^2u}{dy^2}, \qquad (1)$$

$$v\frac{dv}{dy} + \mu v_t \frac{dv_t}{dy} = \nu \frac{d^2v}{dy^2}, \qquad (2)$$

$$v\frac{dt}{dy} + \mu \frac{c_t}{c_p} v_t \frac{dt_t}{dy} = \frac{\lambda}{\rho c_p} \frac{d^2t}{dy^2}. \qquad (3)$$

In writing the motion and energy equations for the components, it is assumed that the transfer of momentum between the particles and the carrying medium is described by Stokes' law and that the heat exchange between them is mainly by conduction. Then for the solid particles

$$\rho_t v_t \frac{dv_t}{dy} = \frac{18\rho\nu (v - v_t)}{d_t^2}, \quad \rho_t v_t c_t \frac{dt_t}{dy} = \frac{6\alpha_t}{d_t}(t - t_t). \qquad (4)$$

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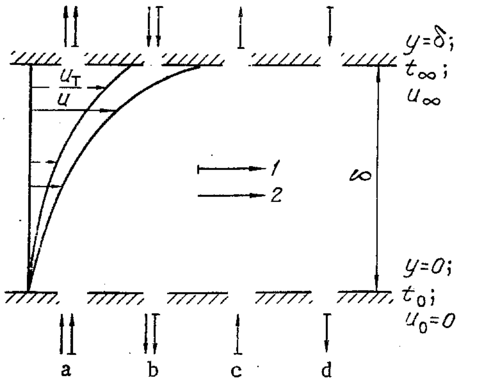

Fig. 1

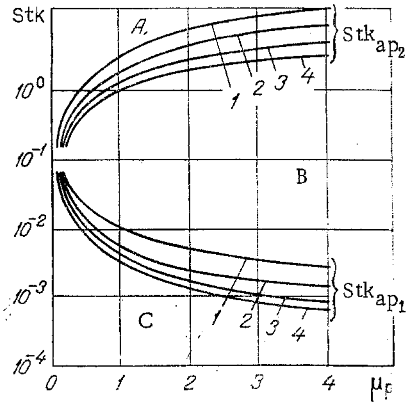

Fig. 2

Fig. 1. Diagram of two-dimensional Couette flow with transverse mass transport: a, b) injection and withdrawal of gas suspension; c, d) ablation and deposition of particles; 1) particles; 2) gas.

Fig. 2. Regions of applicability of approximate models in the withdrawal of a gaseous suspension: A) "single particle"; B) two-component; C) quasiliquid; 1) Re_0 = 5; 2) 10; 3) 15; 4) 20.

The following variables can be used in changing over to dimensionless form:

$$Y=\frac{y}{\delta};\ X=\frac{x}{\delta};\ U=\frac{u}{u_\infty};\ U_t=\frac{u_t-u_{t0}}{u_\infty};\ V_t=\frac{v_t}{v_{t0}};$$

$$T=\frac{t-t_0}{t_\infty-t_0},\ T_t=\frac{t_t-t_{t0}}{t_\infty-t_0},\ Re_0=\frac{v_{t0}\delta}{\nu},\ Re_\infty=\frac{u_\infty\delta}{\nu},$$

$$Stk=\frac{v_{t0}\tau_p}{\delta}=\frac{Re_0}{18}\frac{\rho_t}{\rho}\left(\frac{d_t}{\delta}\right)^2,\ Eu^*=\frac{Eu}{X}=\frac{\tau_0}{\rho u_\infty^2}, \tag{5}$$

$$Nu_w=\frac{\alpha_w\delta}{\lambda},\ Nu_t=\frac{\alpha_t d_t}{\lambda},\ Pr=\nu/a,$$

$$\mu_p=\frac{\beta}{1-\beta}\frac{\rho_t}{\rho}\frac{v_t}{v},\ \mu=\frac{\beta}{1-\beta}\frac{\rho_t}{\rho},$$

where $\alpha_w = q_w/(t_\infty - t_0)$; $\tau_p = \rho_t d_t^2/18\rho\nu$.

The boundary conditions, the character of relative motion, and the possible simplifications in the mathematical description differ significantly for different schemes of motion of the components. Transverse flow without stagnation is examined for injection and withdrawal (by suction) of the gas suspension ($v = v_t$ = const). In ablation and deposition, there is viscous drag exerted on the particles through the height of the bed ($v_t\mu$ = const), there is no gas flow rate in the transverse direction, the mass concentration of the particles is usually very small ($\mu < 1$), and it may be approximately assumed for these conditions that $(1 - \beta) \simeq 1$.

The works [7-11] use the Couette flow model as the basis for studying the possible effect of transverse motion of the solid component on the characteristics of the carrying medium for individual flow schemes. The data in [7-11] makes it possible to qualitatively analyze the nature of the effect of certain characteristics of the solid component, but does not establish the interaction of the flow with the wall and the possibility of using approximate models. To solve these problems, in analyzing particle deposition we will henceforth examine the system of motion and energy equations for the solid component (4) and the flow (1)-(3) with the following boundary conditions:

$$Y=0,\ U=T=0;\ Y=1,\ U=T=1,\ U_t=T_t=0,\ V_t=1. \tag{6}$$

The equation of transverse motion of the solid component, with allowance for (6), can be solved independently of the remaining equations:

$$V_t = 1 - \frac{1}{\mathrm{Stk}}(1 - Y). \quad (7)$$

If we know V_t, then, in a manner similar to [10], we can make the substitution

$$n^2 = 4\mathrm{Re}_0\,\mathrm{Stk}\,V_t\,\mu_0 \quad (8)$$

and reduce the system of motion equations to the form of a modified Bessel equation which can be solved independently of the energy equations:

$$U_t = n\,[c_1 I_1(n) + c_2 K_2(n)] + \frac{\mathrm{Eu}^*\mathrm{Re}_\infty}{\mathrm{Re}_0\mu_0}, \quad (9)$$

$$U = \frac{1}{2}n^2\,[-c_1 I_2(n) + c_2 K_2(n)] + \frac{\mathrm{Eu}^*\mathrm{Re}_\infty}{\mathrm{Re}_0\mu_0} + \frac{u_{t\delta}}{u_\infty}, \quad (10)$$

$$\mathrm{Eu}^* = -\frac{\mathrm{Re}_0\mu_0}{\mathrm{Re}_\infty} n_\delta\,[c_1 I_1(n_\delta) + c_2 K_2(n_\delta)]. \quad (11)$$

Introducing the notation

$$\omega = n\left(\frac{\alpha_t d_t}{3\lambda}\right)^{1/2} \simeq n\sqrt{2/3} \quad (12)$$

(since $\mathrm{Nu}_t = 2$ and $c_p/c_t \simeq 1$, $\mathrm{Pr} \simeq 2/3$), we can obtain a solution for the energy equations

$$T_t = \omega\,[c_3 I_1(\omega) + c_4 K_1(\omega)] + \frac{\mathrm{Nu}_w}{\mathrm{Re}_0\mathrm{Pr}\mu_0}\frac{c_p}{c_t}, \quad (13)$$

$$T = \frac{1}{2}\omega^2\,[-c_3 I_2(\omega) + c_4 K_2(\omega)] + \frac{\mathrm{Nu}_w}{\mathrm{Re}_0\mathrm{Pr}\mu_0}\frac{c_p}{c_t} - \frac{t_0 - t_{t0}}{t_\infty - t_0}. \quad (14)$$

The integration constants c_1, c_2, c_3, and c_4 are determined from the boundary conditions (6). The rate of heat transfer from the flow to the wall with boundary conditions (6) and low radiative transfer is determined by the expression

$$\mathrm{Nu}_w = -\omega_\delta\,[c_3 I_1(\omega_\delta) + c_4 K_1(\omega_\delta)]\,\mathrm{Re}_0\mathrm{Pr}\mu_0 c_t/c_p. \quad (15)$$

Using the relations obtained and tabulated values of first- and second-order Bessel functions of the first and second kind, we can calculate the velocity and temperature fields and the rates of thermal and hydrodynamic interaction of the flow with the wall for different parameters of the gas suspension flow in the case of particle deposition.

For particle ablation, the solution is obtained in a manner similar to [10].

Again using the Couette flow model as a basis, a study was made of the velocity and temperature fields for the cases of the motion of a gas suspension flow through a boundary layer from a wall [9] and to a wall [8]. For the cases of injection and withdrawal of the gas suspension, the initial system of equations consisted of the motion and energy equations of the solid and gaseous components, obtained from (1)-(3). In contrast to the dimensionless variables (5) introduced earlier, $U_t = u_t/u_\infty$ and $T_t = (t_t - t_0)/(t_\infty - t_0)$. It should also be noted that at $c_t/c_p \simeq 1$, $\mathrm{Nu}_t = 2$ (this is acceptable in studying gas suspension flows with moderate velocities of relative particle motion), similitude of the velocity and temperature fields of the components is observed. The solution for injection of the gas suspension is obtained with the boundary conditions

$$Y = 0,\ U = U_t = 0,\ T = T_t = 0;\ Y = 1,\ U = 1,\ T = 1. \quad (16)$$

For the case of withdrawal of the gas suspension with the boundary conditions

$$Y = 0,\ U = T = 0;\ Y = 1,\ U = U_t = T = T_t = 1 \quad (17)$$

we can obtain

$$U=\frac{(1-k_1\,\mathrm{Stk})[1-\exp(k_1Y)]-(k_1\exp k_1/k_2\exp k_2)\times(1-k_2\,\mathrm{Stk})(1-\exp(k_2Y))}{(1-k_1\,\mathrm{Stk}-\exp k_1)-(k_1\exp k_1/k_2\exp k_2)\times(1-k_2\,\mathrm{Stk}-\exp k_2)}, \quad (18)$$

$$U_t=\frac{[1-k_1\,\mathrm{Stk}-\exp(k_1Y)]-(k_1\exp k_1/k_2\exp k_2)\times[1-k_2\,\mathrm{Stk}-\exp(k_2Y)]}{(1-k_1\,\mathrm{Stk}-\exp k_1)-(k_1\exp k_1/k_2\exp k_2)\times(1-k_2\,\mathrm{Stk}-\exp k_2)}, \quad (19)$$

$$k_{1,2}=\frac{1-\mathrm{Re}_0\,\mathrm{Stk}}{2\,\mathrm{Stk}}\left[1\pm\sqrt{1+\frac{4\,\mathrm{Re}_0\,\mathrm{Stk}\,(1+\mu_p)}{(\mathrm{Re}_0\,\mathrm{Stk}-1)^2}}\right]. \quad (20)$$

It is not hard to show that the expressions obtained for the velocity fields can be used to obtain solutions for the case of injection (withdrawal) when the concentration of solid particles is vanishingly small ($\mu_p \to 0$). The velocity distribution of the gas in this case is similar to the distribution in the case of homogeneous injection (withdrawal):

$$U=\frac{1-\exp(\pm\,\mathrm{Re}_0Y)}{1-\exp(\pm\,\mathrm{Re}_0)}. \quad (21)$$

Accordingly, for the solid component in the case of injection

$$U_t=\frac{1-\exp(\mathrm{Re}_0Y)+\mathrm{Re}_0\mathrm{Stk}\,[1-\exp(-Y/\mathrm{Stk})]}{(1+\mathrm{Re}_0\mathrm{Stk})[1-\exp(-\mathrm{Re}_0)]} \quad (22)$$

and in withdrawal of the gas suspension

$$U_t=\frac{1-\exp(-\mathrm{Re}_0Y)+\mathrm{Re}_0\mathrm{Stk}\left[1-\exp\left(-\frac{1+\mathrm{Re}_0\mathrm{Stk}-Y}{\mathrm{Stk}}\right)\right]}{(1+\mathrm{Re}_0\mathrm{Stk})[1-\exp(-\mathrm{Re}_0)]}. \quad (23)$$

In the case of injection or withdrawal at $\mathrm{Stk} \to 0$, the particles and gas move at nearly the same speed, i.e., there is quasihomogeneous injection (withdrawal):

$$U=U_t=\frac{1-\exp[\pm\mathrm{Re}_0(1+\mu_p)Y]}{1-\exp[\pm\,\mathrm{Re}_0(1+\mu_p)]}. \quad (24)$$

The resulting relations (7)-(15) and (18)-(24) can be used to calculate velocity and temperature fields in the boundary region for different schemes of motion of the components. However, it is usually integral characteristics of the dynamic and thermal interaction of the flow with the wall that are of the greatest interest. The intensity of such interaction can be calculated from the velocity and temperature fields, determined with allowance for the considerable heterogeneity of the flow ((18)-(19)), and can also be evaluated from approximate relations obtained either with the assumption of a quasihomogeneous mixture (24) or for a "single particle" (21). Obviously, comparing the results of calculations with these relations would make it possible to determine the limits of applicability of the corresponding approximations in calculating the interaction of the flow with the wall.

As an example, Fig. 2 shows the results of a similar analysis for the case of withdrawal of a gas suspension. Curves 1-4 correspond to a 5% deviation of the shear stresses on the wall, calculated from the approximate relations, from the values obtained with allowance for the heterogeneity of the flow. Region A corresponds to the absence of an effect by the particles on the velocity and temperature fields of the gas ("single particle"); region C corresponds to the quasihomogeneous approximation; region B characterizes the case where the heterogeneity of the particles exerts a substantial effect. The relations $\mathrm{Stk}_{ap1} = f_1(\mathrm{Re}_0, \mu_p)$ and $\mathrm{Stk}_{ap2} = f_2(\mathrm{Re}_0, \mu_p)$ represents the limits of application of the corresponding approximations. Region A has the highest values for thermal and velocity slip of the compo-

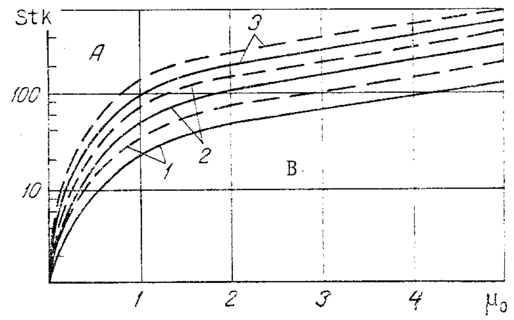

Fig. 3. Limits of application of the "single particle" approximation (region A) in particles deposition (solid lines) and ablation (dashed lines): 1) Re_0 = 5; 2) 10; 3) 15; B) region of significant effect of heterogeneity.

nents is vanishingly small for region C, but the effect of the impurities on the velocity and temperature profiles of the gas component is greatest. At very low concentrations ($\mu_p \leqslant 10^{-1}$, Fig. 2), the effect of the particles on flow characteristics is so slight that the relations for a quasihomogeneous fluid give the same results as for a homogeneous gas. The results obtained permit us to quantitatively determine the limits of application of the respective approximations. For example, in turbulent pipe flow, if we take the transverse velocity of the suspension (of a pulsative nature, say) to be equal to the dynamic velocity then $Re_0 \equiv v_0\delta/\nu \simeq 10$ on the boundary of the viscous sublayer. Then, in accordance with Fig. 2, the quasihomogeneous fluid approximation can be used at $\mu_p \simeq 2$ for $Stk < 1.8\cdot10^{-3}$ (which, at $\rho_t/\rho \simeq 2000$, is equivalent to $d_t/\delta \leqslant 1.3\cdot10^{-3}$). Given the same parameters, the "single particle" approximation can be used for the cases Stk > 5 (equivalent to $d_t/\delta \geqslant 6.7\cdot10^{-2}$). In both cases, an increase in the flow-rate concentration leads to an increase in the size of the zone $Stk_{ap_1} < Stk < Stk_{ap_2}$ in which use of the approximations would be incorrect.

Similar laws are found in analyzing the temperature fields from heat-exchange conditions on the wall.

Analysis of the results obtained shows that the "smallness" of μ_p or Stk is not a sufficient condition for substantiated selection of an approximate model in calculating the boundary region in dust-laden flows, and that the deviation may be substantial with an increase in concentration in evaluating heat flows or shear stresses from the approximate relations (at μ_p = 0.2, e.g., the error may be about 30%; at μ_p = 1, it may be about 75%).

For the case of injection of a gas suspension, the region of effect of heterogeneity with regard to Stk (at Re_0 and μ_p = idem) is approximately two orders broader than in the case of withdrawal. This is a consequence of the significantly greater thermal and velocity slip of the particles in direct proximity to the wall (Y = 0) for the conditions of gas suspension injection.

A similar analysis for the cases of ablation and deposition of particles from the conditions of friction and heat exchange at the wall (Fig. 3) was done only for the "single particle" approximation ($\mu \to 0$). The results of the calculation using the relations obtained were compared with the velocity and temperature distribution in a Couette flow without transverse motion of the components. Since there is no companion motion of the gas in the transverse direction (G = 0) in ablation and deposition, the notion of a flow-rate concentration μ_p loses significance and is not used. Instead, we assign the true mass concentration of the particles μ_0 at the flow inlet as the determining parameter. Conducting an analysis with the relations (9)-(11), (13)-(15) is a laborious task, since choosing among the variants necessarily involves repeated calculations of Bessel functions. Thus, the analysis was performed on the basis of a numerical solution of the initial system on a BESM-4 computer by the Kutta-Merson method. A minimal Stokes number follows from the condition of constancy of the flow rate of the solid component in the transverse directions, i.e., the absence of accumulation of particles in the flow, and, in accordance with boundary conditions (6), $Stk_{min} \geqslant 1$. Since particle drag occurs in the present case, the dependence of Stk_{ap_2} on the Reynolds number will be different than in the case of injection and withdrawal of the suspension. The larger Re_0 (calculated for the transverse velocity), the greater the phase interaction and the more substantial the effect of the particles on the gas.

Thus, with an increase in Re_0, the inertial parameter of the impurity for the "streaming" should be greater and, in accordance with Fig. 3, substantiated selection of computation models is essential — even for flows with a low dust content ($\mu < 1$ [12]). For Re_0 = idem and μ_0 = idem, the absolute slip $(U - U_t)$ at $Y = 0$ is greater in deposition than for the case of ablation, so the region of the effect of heterogeneity will be broader.

The results obtained show that the selection of models for calculating processes in the boundary zone of gas suspension flows cannot be arbitrary, and the most commonly-used models — "single-particle" and "quasihomogeneous fluid" — have strictly defined limits of application (see Figs. 2 and 3). These limits can be refined for specific conditions by allowing for additional force factors, features of flow at high velocities, compressibility of the gas, and the effect of temperature.

NOTATION

ρ, density; ν, kinematic viscosity; λ, thermal conductivity; c_p, c_t, heat capacity of the gas and the material of the solid particles; μ_p, μ, β, flow-rate mass, true mass, and volume concentrations; d_t, particle diameter; δ, distance between plates; y, transverse coordinate; v, u, transverse and longitudinal velocities; t, temperature; τ_0, shear stresses on the wall; I_1, I_2, K_1, K_2, modified first- and second-order Bessel functions of the first and second kind. Indices: 0, ∞, parameters on the bottom and top plates; t, solid component.

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